

**CISC 603-51- A-2021/SUMMER - THEORY OF COMPUTATION**

**Assignment - 1**

**Mathematical Foundations**

**Languages, Grammars, Automata**

**By,**

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* 1. Mathematical Preliminaries and Notation

#1

S1={2,3,5,7}

S2={2,4,5,8,9}

U = {1:10} implies elements from 1 to 10 all integers = {1,2,3,4,5,6,7,8,9,10}

Compute 1 U S2

Note: Complement implies universe – the set

(U - 1) U S2

(U - 1)

{1,2,3,4,5,6,7,8,9,10} – {2,3,5,7} = {1,4,6,8,9,10}

(U - 1) U S2 🡪 Union is the set of all the elements in both sets without repititon.

{1,2,4,5,6,8,9,10}

#2

S1={2,3,5,7} S2={2,4,5,8,9}

S1 X S2

{(2,2),(2,4), (2,5),(2,8) (2,9), (3,2),(3,4), (3,5),(3,8) (3,9), (5,2),(5,4), (5,5),(5,8) (5,9), (7,2),(7,4), (7,5),(7,8) (7,9) }

S2 X S1

S2={2,4,5,8,9} S1={2,3,5,7}

{(2,2),(2,3), (2,5),(2,7), (4,2),(4,3), (4,5),(4,7), (5,2),(5,3), (5,5),(5,7), (8,2),(8,3), (8,5),(8,7), (9,2),(9,3), (9,5),(9,7)}

#3

|S ∩ T|

First compute S ∩ T is the list of all the elements common on both S and T i.e., {2,6,8}

|S ∩ T| = 3

Now compute S U T which is the list of all elements from S and T without repition

{2,4,5,6,8}

| S U T | = 5

|S ∩ T| + | S U T | = 3 + 5 = 8

#4

| S U T | = |S| + |T|

For Example

S= {2,4,6} T = {8, 10,12}

S U T = {2,4,6,8,10,12}

|S U T| = 6 🡪 Number of elements in S U T

|S| = 3 🡪 Number of elements in set S

|T| = 3 🡪 Number of elements in set T

6 = 3 + 3

These types of sets are called Disjoint sets 🡪 sets which have no elements in common.

#5

S – T = S ∩

X ϵ |S - T|

Since S -T will eliminate all the elements common in S and T and all the elements in T, therefore

X ϵ S and X ∉ T

X ϵ S and X ϵ Tc

X ϵ S ∩

#6

Prove De Morgan’s law

X ϵ (S1 U S2)c

Which means X ∉ (S1 U S2)

X ϵ S1c ∩ S2c

X ∉ S1 and X ϵ S2c

Consider RHS

X ϵ S1c ∩ S2c

X ∉ S1 and X ∉ S2c

Implies X ϵ (S1 U S2)c

Hence proved.

#7

S1 ⊆ S2

⊆

Let X ϵ = X does not belongs to S2

Since (S1)c is a subset or equal to S2c

Therefore xcannot belong to S1

⊆

#8

S1 U S2 = S1 ∩ S2

Let x ϵ (S1 ∩ S2) = X ϵ S1 and x ϵ S2

Simply S1 union S2 is a set of all elements in S1 and S2

S1 intersection S2 is only the common elements in S1 and S2

Both these statements can be true only when both the sets S1 and S2 contains same identical elements, that is both S1 and S2 are equal.

1.2 Three Basic Concepts

#1

wwRw where w = aabbab

wR = babbaa 🡪 is the reverse of w

aabbab**babbaa**aabbab

Find the string aab in the above string

**aab**babbabbaa**aab**bab

The answer is Two

#2

|un| = n|u|

For n = 1

|u| = |u|

For n =2

|u2| = |u u| = |u| + |u| = 2 |u|

Generalizing the above equation

|un+1| = |un| + |u|

| un+1| = |un-1|+ |u| + |u|

| un+1| = (n+1)|u|

#3

(aR) = a

(wa)R = awR

Prove that : (uv)R = vRuR

(uw)R = (uva)R

= a vR uR

#4

wR = (w1. w2.w3 …. wn)R

=wn(w1.w2.w3 ….. wn-1)R

=wn wn-1(w1 w2 w3…..wn-2)R

= wn wn-1 wn-2 ….. w3 w2 w1

(WR)R

= (wn wn-1 wn-2 ….. w3 w2 w1)R

=w1 (wn wn-1 wn-2 ….. w3 w2)R

= w1 w2 (wn wn-1 wn-2 ….. w3)R

= w1. w2.w3 …. Wn = w

#5

aaaabaaaa = aa aa baa aa belongs to L = {ab, aa, baa}

baa aa a baa aab = cannot be decomposed into strings in L

baa aa ab aa = can be decomposed and are in L